

Barem clasa a VII-a (OLM 2017-etapa locală)

Subiectul I. (7 puncte)

a) $S = \frac{2018}{1 \cdot 3} + \frac{2018}{3 \cdot 5} + \dots + \frac{2018}{2015 \cdot 2017} = 1009 \cdot \left(\frac{2}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \dots + \frac{2}{2015 \cdot 2017} \right) =$ (2 puncte)

$= 1009 \cdot \left(1 - \frac{1}{2017} \right) = 1009 \cdot \frac{2016}{2017} \Rightarrow 2017 \cdot S = 1009 \cdot 2016 \in N$ (2 puncte)

b) $\frac{1}{2^2} < \frac{1}{1 \cdot 2} ; \frac{1}{3^2} < \frac{1}{2 \cdot 3} ; \frac{1}{4^2} < \frac{1}{3 \cdot 4} ; \dots ; \frac{1}{2017^2} < \frac{1}{2016 \cdot 2017}$ (2 puncte)

Adunând relațiile membru cu membru, obținem inegalitatea din enunț (1 punct)

Subiectul II. (7 puncte)

$\frac{a}{2b+3c} = \frac{2b}{a+3c} = \frac{3c}{a+2b} = \frac{a+2b+3c}{2(a+2b+3c)} = \frac{1}{2}$ (2 puncte)

$\left. \begin{array}{l} 2a - 2b - 3c = 0 \\ a - 4b + 3c = 0 \\ a + 2b - 6c = 0 \end{array} \right\} \Rightarrow a = 2b ; c = \frac{2b}{3}$ (2 puncte)

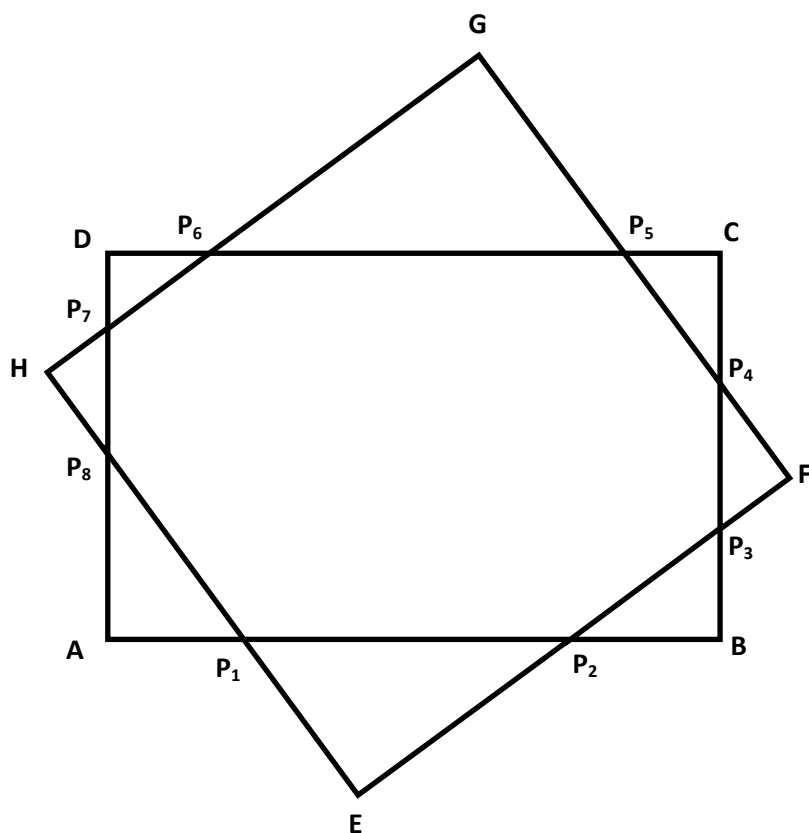
$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{b} \in N^* \Rightarrow b = 1 \text{ sau } b = 3$ (2 puncte)

Pentru $b=1$ obținem $\frac{1331}{27}$, iar pentru $b=3$ obținem 11, deci valoarea minimă este 11 (1 punct)

Subiectul III. (7 puncte)

Desen

(1 punct)



$$\left. \begin{aligned} \sphericalangle AP_1P_8 &\equiv \sphericalangle EP_1P_2 \text{ (op. la v.)} \\ \sphericalangle P_1AP_8 &\equiv \sphericalangle P_1EP_2 \text{ (drepte)} \end{aligned} \right\} \Rightarrow \Delta AP_1P_8 \sim \Delta EP_1P_2 \Rightarrow \frac{AP_1}{P_1E} = \frac{P_1P_8}{P_1P_2} \quad (3 \text{ puncte})$$

$$\text{Analog } \frac{EP_2}{P_2B} = \frac{P_1P_2}{P_2P_3}, \frac{BP_3}{P_3F} = \frac{P_2P_3}{P_3P_4}, \frac{FP_4}{P_4C} = \frac{P_3P_4}{P_4P_5}, \dots, \frac{HP_8}{P_8A} = \frac{P_7P_8}{P_8P_1} \quad (2 \text{ puncte})$$

$$\frac{AP_1}{P_1E} \cdot \frac{EP_2}{P_2B} \cdot \frac{BP_3}{P_3F} \cdot \frac{FP_4}{P_4C} \cdot \frac{CP_5}{P_5G} \cdot \frac{GP_6}{P_6D} \cdot \frac{DP_7}{P_7H} \cdot \frac{HP_8}{P_8A} = \frac{P_1P_8}{P_1P_2} \cdot \frac{P_1P_2}{P_2P_3} \cdot \frac{P_2P_3}{P_3P_4} \cdot \frac{P_3P_4}{P_4P_5} \cdot \frac{P_4P_5}{P_5P_6} \cdot \frac{P_5P_6}{P_6P_7} \cdot \frac{P_6P_7}{P_7P_8} \cdot \frac{P_7P_8}{P_8P_1} = 1 \quad (1 \text{ punct})$$

Subiectul IV. (7 puncte)

Desen (1 punct)

$$\left. \begin{aligned} B \text{ este mijlocul lui } [DF] \\ BE \parallel DM \end{aligned} \right\} \Rightarrow [BE] \text{ linie mijlocie în } \Delta MDF \quad (1 \text{ punct})$$

$$\Rightarrow MD = 2BE \Rightarrow MD = AB = DC. \quad (1 \text{ punct})$$

$$\text{În } \Delta MFC, D \text{ mijlocul lui } [MC] \Rightarrow A_{\Delta MDF} = A_{\Delta FDC} \quad (1 \text{ punct})$$

$$\text{În } \Delta DCF, B \text{ mijlocul lui } [DF] \Rightarrow A_{\Delta FBC} = A_{\Delta BCD} \quad (1 \text{ punct})$$

$$\text{În } \Delta ABD, E \text{ mijlocul lui } [AB] \Rightarrow A_{\Delta ADE} = A_{\Delta BED} \quad (1 \text{ punct})$$

$$\text{Din (1) și (2)} \Rightarrow A_{\Delta MDF} = 2 \cdot A_{BCD} = 4 \cdot A_{AED}. \quad (1 \text{ punct})$$